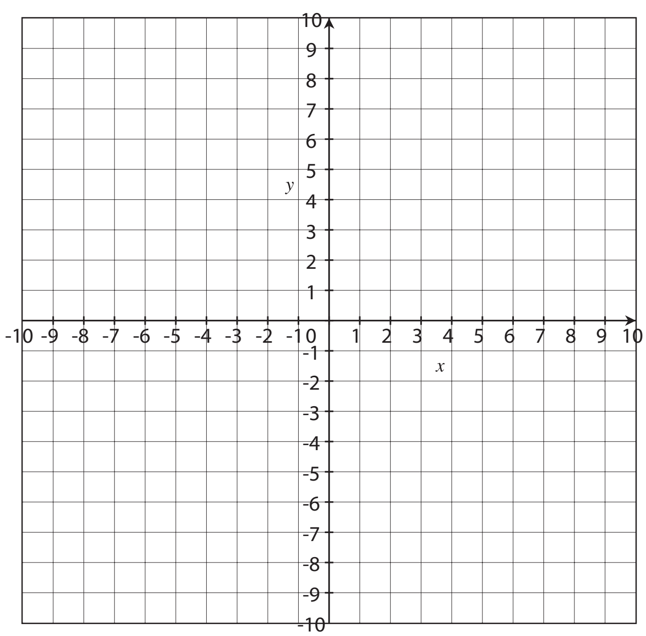
Scaffolded Practice 2.1.1

Example 1

Graph the solution set for the linear equation $-3x + y = -2$.

1. Solve the equation for y.
2. Make a table. Choose at least 3 values for x and find the corresponding values of y using the equation.

3. Plot the ordered pairs on the coordinate plane.



4. Connect the points by drawing a line through them. Use arrows at each end of the line to show that the line continues indefinitely in each direction. This represents all of the solutions for the equation.

continued

Example 2

Graph the solution set for the equation $y = 3^x$.

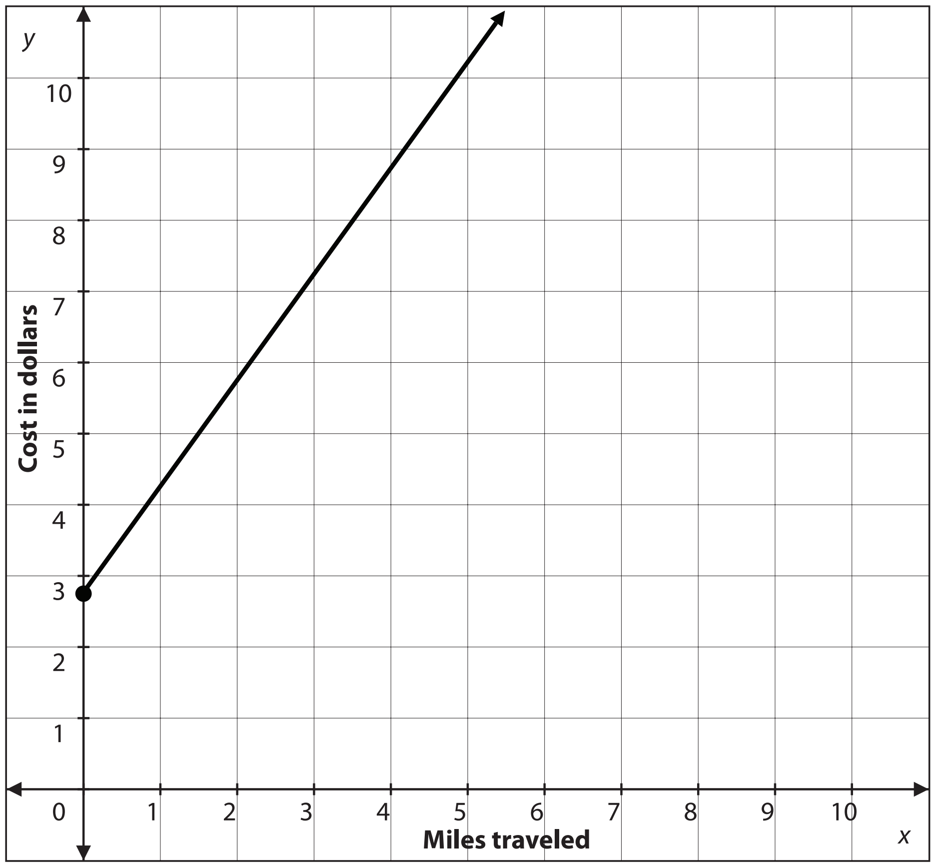
Example 3

The Russell family is driving 1,000 miles to the beach for vacation. They are driving at an average rate of 60 miles per hour. Write an equation that represents the distance remaining in miles and the time in hours they have been driving, until they reach the beach. They plan on stopping 4 times during the trip. Draw a graph that represents all of the possible distances and times they could stop on their drive.

Scaffolded Practice 2.4.1

Example 1

A taxi company in Atlanta charges $2.75 per ride plus $1.50 for every mile driven. Determine the key features of this function.



1. Identify the type of function described.

2. Identify the intercepts of the graphed function.

continued

3. Determine whether the graphed function is increasing or decreasing.

4. Determine where the function is positive and negative.

5. Determine the relative minimum and maximum of the graphed function.

6. Identify the domain of the graphed function.

7. Identify any asymptotes of the graphed function.

continued

Example 2

A pendulum swings to 90% of its height on each swing and starts at a height of 80 cm. The height of the pendulum in centimeters, *y*, is recorded after *x* number of swings. Determine the key features of this function.

Number of swings (*x*)	Height in cm (*y*)
0	80
1	72
2	64.8
3	58.32
5	47.24
10	27.89
20	9.73
40	1.18
60	0.14
80	0.02

Example 3

A ringtone company charges $15 a month plus $2 for each ringtone downloaded. Create a graph and then determine the key features of this function.

Scaffolded Practice 2.1.4

Example 1

Evaluate $f(x) = 4x - 7$ over the domain {1, 2, 3, 4}. What is the range?

1. To evaluate $f(x) = 4x - 7$ over the domain {1, 2, 3, 4}, substitute the values from the domain into $f(x) = 4x - 7$.

2. Evaluate $f(1)$.

3. Evaluate $f(2)$.

4. Evaluate $f(3)$.

5. Evaluate $f(4)$.

6. Collect the set of outputs from the inputs.

continued

Example 2

Evaluate $g(x) = 3^x + 1$ over the domain {0, 1, 2, 3}. What is the range?

Example 3

Raven started an online petition calling for more vegan options in the school cafeteria. So far, the number of signatures has doubled every day. She started with 32 signatures on the first day. Raven's petition can be modeled by the function $f(x) = 32(2)^x$. Evaluate $f(3)$ and interpret the results in terms of the petition.

Prerequisite Skills

This lesson requires the use of the following skills:

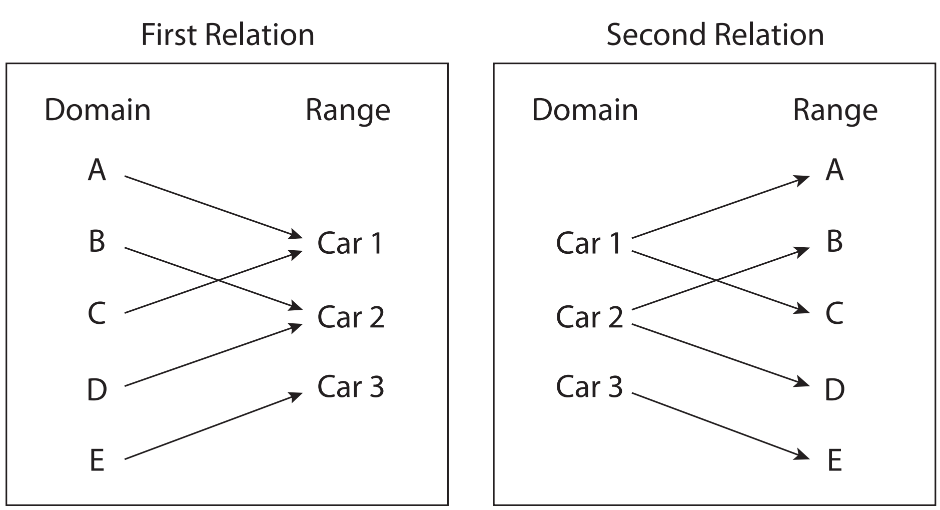
- substituting values for variables
- following the order of operations
- evaluating negative exponents

Introduction

Earlier we saw that the graph of $y = f(x)$ is the set of all solutions to the function f. We also found that when $f(x) = g(x)$, the functions f and g share a common solution and the graphs of these functions will intersect at that point. In this lesson, we learn what defines a function and the properties of a function.

Before we define what it is to be a function, we must first define a relation. A **relation** is a relationship between two sets of data. If you pair any two sets of data, you create a relation between the sets. For example, you could define a relation between the neighbors on your block and the cars they drive such that if you are given a neighbor's name, you can properly assign that person to his or her car. And, you could define another relation between the same sets such that if you are given a car's make and model, you can assign that car to the correct neighbor.

Remember that a function is a special relation in which each input is mapped to only one output. So, as with the first example, say we are given a name of a neighbor and we assign that neighbor to a make and model of a car. Even if some of your neighbors drive the same make and model car, each person's name is assigned to only one type of car. This relation is a function. But, what if we go the other way and look at the make and model of the cars and map them to the neighbors who drive them? Then a particular make and model of a car may be assigned to two or more neighbors. Both relations are pictured below:

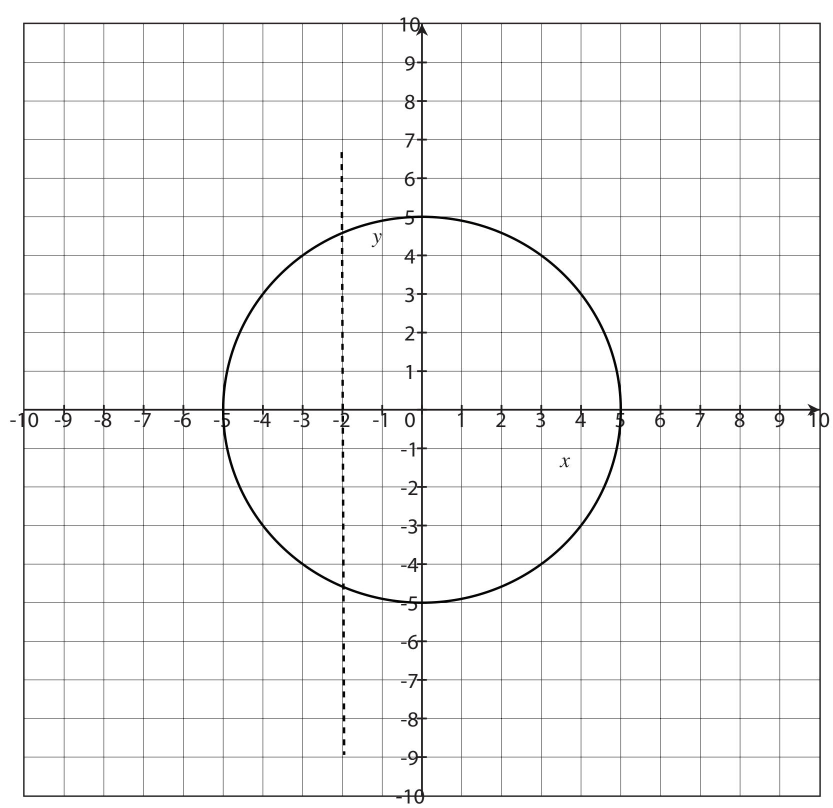


In the first relation, neighbors A and C are both mapped to Car 1 and neighbors B and D are mapped to Car 2. But what is important is that all the neighbors A–E are each mapped to only one car; therefore, this relation is a function. In the second relation, we see Car 1 is mapped to neighbors A and C, and Car 2 is mapped to neighbors B and D. The second relation is not a function.

We call the set of all potential inputs the **domain** of the function. We call the set of potential outputs the **range** of the function. So a function f takes an element x from the domain and creates $f(x)$, an element in the range. It is important to understand that $f(x)$ is strictly one value; each element in the domain of a function can be mapped to exactly one element in the range. That is, for every value of x, there is exactly one value of $f(x)$.

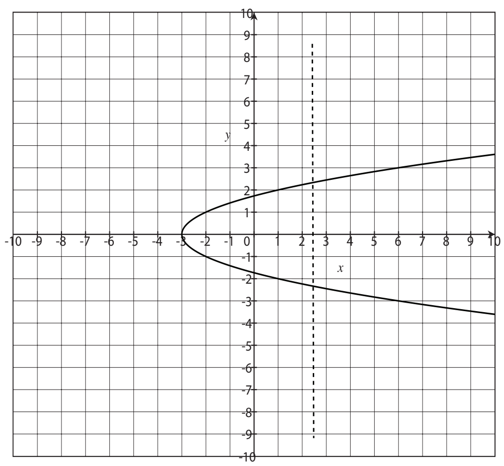
One way to determine whether a relation is a function is to graph the relation and perform a vertical line test. A vertical line in the coordinate plane is described by $x = a$, where a is the value of x where the line crosses the x-axis. So, if a vertical line crosses a graph at two unique points, this implies the graphed relation has two unique values for $f(a)$. In other words, if the vertical line crosses the graph in only one place, the graph is a function. If the line crosses the graph in two or more places, it is not a function.

Take a look at the following graph of a complete circle. If we draw a vertical line where $x = -2$, we see that the line crosses the graph at two points. This means the relation that describes this graph will map $x = -2$ to more than one value. Therefore, the graph is not a function.

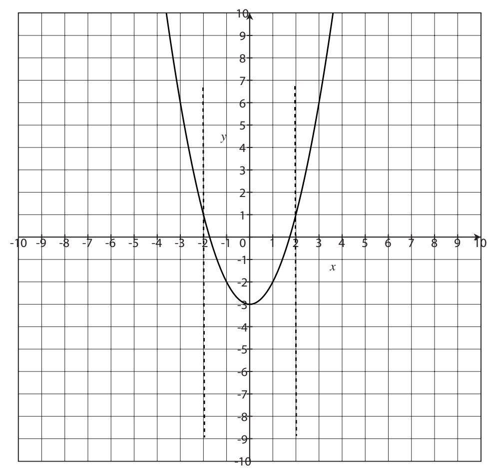


The following graphs are very similar, but the graph on the left fails the vertical line test and the graph on the right passes the vertical line test. Therefore, the graph on the left is not a function and the graph on the right is a function.

Is not a function:



Is a function:



Key Concepts

- The domain is the set of *x*-values that are valid for the function.
- The range is the set of *y*-values that are valid for the function.
- A function maps elements from the domain of the function to the range of the function.
- Each *x* in the domain of a function can be mapped to one *f*(*x*) in the range only.
- If an element in the domain maps to more than one element in the range, then the relation is not a function.
- To create a mapping, list the domain in one column and the range in a second column. Then draw lines that match the elements in the domain to the corresponding elements in the range.
- In the mapping, if one line goes from one *x*-value to only one *y*-value, then the relation is a function.
- If more than one line goes from an element in the domain (*x*-values) to multiple elements in the range (*y*-values), then the relation is not a function.
- The vertical line test can also be used to determine if a relation is a function. If you pass an imaginary vertical line across the graph, look to see if the line ever crosses more than one point on the graph at a time.

- If a vertical line that sweeps across the graph crosses only one point on the graph, then the relation is a function.
- If a vertical line that sweeps across the graph crosses more than one point on the graph at the same time, then the relation is not a function.

Common Errors/Misconceptions

- confusing domain and range
- thinking that if a range value repeats or is the same for a different value in the domain that the relation is not a function (different *x*-values have the same *y*-value)